Joint modelling of international yield curves[1]

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Abstract

In this paper we propose a new approach to modelling and estimating yield curves across multiple currency areas. The idea is that one economy is assumed to be the 'cardinal' economy which affects the evolution of the yield curves in the other markets. The adopted methodology is inspired by the 3-factor Nelson-Siegel yield curve model and seems to capture the yield curve cross-sectional dynamics as well as the time-series dynamics in an acceptable way.

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1 Introduction

International fixed income investors and risk managers require models that facilitate simultaneous evolution of yield curves in several currency areas. This is true for investment banks, pension funds and public wealth managers such as central banks. In the context of portfolio optimisation, such a model will ensure that expected return distributions can be calculated in a manner that is consistent with historical correlations across multiple currencies; in addition, the shape, location and tail risk measures of the return distributions can be estimated, to serve as input to markowitz inspired portfolio optimisation. For risk management purposes, for example, stress testing of international bond portfolios can be conducted within a simulation based exercise where effects in one currency area affect the yield curve evolution in the other modelled currency areas.

This paper creates a linkage between international yield curves by expanding the Nelson-Siegel (1987) three factor representation. From a modelling perspective it is proposed to identify one country as being the "cardinal" economy, among the set of modelled yield curves, and to model this yield curve segment using the Nelson-Siegel framework. The remaining countries' yield curves are then modelled as spreads to the "cardinal" curve. Using principal-component analysis the paper shows how a loading structure can be parameterised for each of the non-cardinal countries. The joint multi-currency model is estimated on data from US, Germany and Japan and the in-sample fit is evaluted.

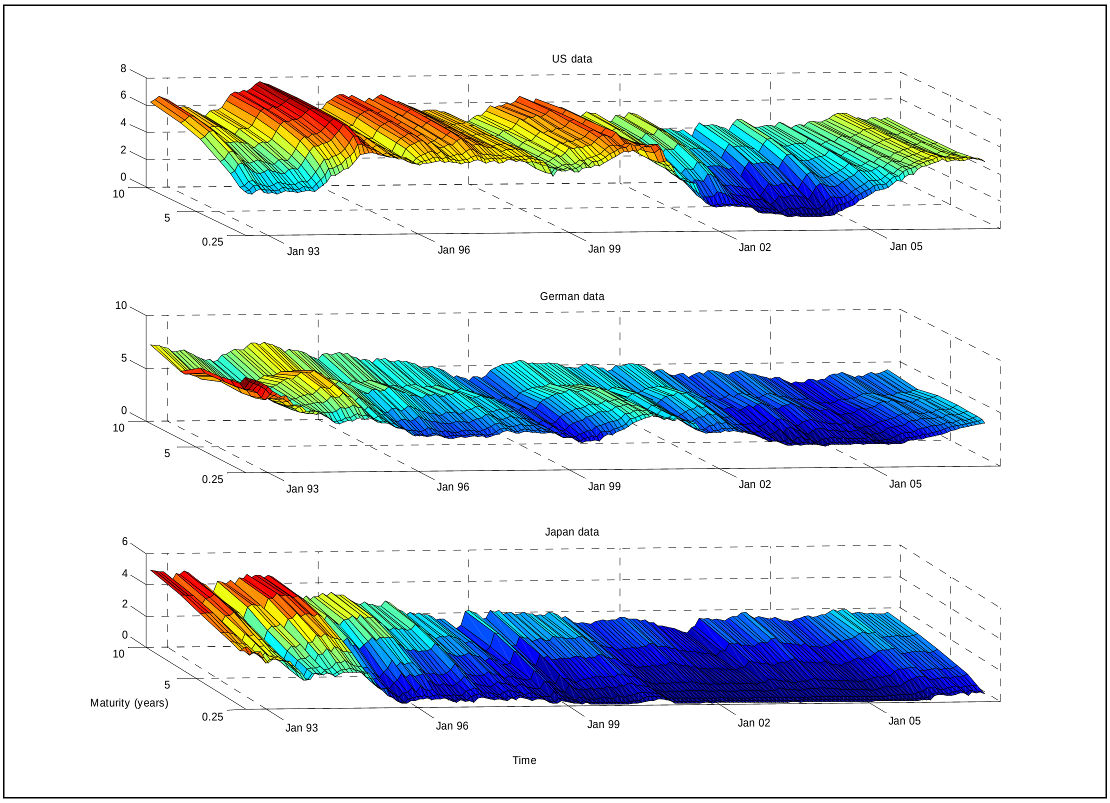
A series of papers offer other expansions of the Nelson-Siegel framework; see among others, Diebold-Li (2006) who, for forecasting purposes, suggest modelling the time-series evolution of the underlying yield curve factors in a VAR model, thereby adding a time-dimension to the original Nelson-Siegel model; Using a state-space model Diebold-Li-Auroba (2006) integrates macro economic variables into the Diebold-Li model; following Diebold-Li (2006) and Diebold-Li-Auroba (2006), Bernadell-Coche-Nyholm (2005) estimates a state-space representation of the Nelson-Siegel model, however, in addition they add regime-switches and macro economic variables as explanatory variables; Nyholm-Rebonato (2007) compares the performance of the regime-switching model of Bernadell et al (2005) to a semi-parametric yield curve model suggested by Rebonato-Mahal-Joshi-Bucholz-Nyholm (2005). None of the above referred papers cast the Nelson-Siegel model in an international context. This is, however, done by Diebold-Li-Yue (2006). They suggest a multi-currency yield curve model composed of two layers. The first layer is the Nelson-Siegel model at the individual country levels. Then, in a second step, the structure for the global yield curve factors, and their dynamic evolution, are found by treating the single country yield curve factors as observed variables. In a sense, this second step is akin to performing a factor analysis on the set of extracted Nelson-Siegel yield curve factors at the individual country level. Finally, in a multi currency context Koivu-Nyholm-Stromberg (2007) show how extracted Nelson-Siegel yield curve factors, link to, and can help predict the future evolution of foreign exchange rates for long horizons.

The rest of the paper is organised as follows. Section 2 presents the data. Section 3 outlines the general modelling framework and presents a particular parameterisation for the modelled non-cardinal countries. Section 4 presents the estimated model and elaborates on the in-sample fit. Section 5 concludes and suggests directions for future research.

2 The Data

Yield curve data covering the period from October 1992 to May 2007, for US, Germany and Japan is used in the analysis. The data is sampled at a monthly frequency giving a total of 177 time series observations for each of the maturities observed at the {0.5, 1, 2, 3, 5, 7, 10} year segments of the curves.

Figure 1: The data



The data is presented in Figure 1. Surface plots are produced to illustrate how the whole yield curve evolves over time. Below the mean, standard deviation and autocorrelation is reported to further illustrate the properties of the yield curve data.

Table 1: Summary statistics for the yield curve data

Maturity	0.50 Y	1Y	2Y	3Y	5Y	7Y	10Y
			US yields				
mean	4.02	4.14	4.45	4.64	4.95	5.17	5.30
std dev	1.62	1.54	1.46	1.35	1.16	1.06	0.97
min	0.93	1.00	1.22	1.50	2.24	2.80	3.28
max	6.40	6.90	7.32	7.43	7.49	7.54	7.66
A(1)	0.99	0.99	0.99	0.98	0.98	0.98	0.97
A(2)	0.98	0.97	0.96	0.96	0.94	0.94	0.93
A(3)	0.96	0.95	0.94	0.93	0.91	0.90	0.89
A(12)	0.62	0.63	0.63	0.62	0.60	0.59	0.57
				German yields			
mean	3.66	3.75	3.95	4.15	4.51	4.80	5.01
std dev	1.40	1.28	1.21	1.17	1.14	1.16	1.14
min	1.84	1.88	2.01	2.16	2.49	2.78	3.08
max	8.73	7.97	7.78	7.65	7.47	7.37	7.40
A(1)	0.96	0.96	0.96	0.96	0.96	0.97	0.97
A(2)	0.91	0.92	0.91	0.92	0.93	0.94	0.95
A(3)	0.87	0.88	0.87	0.87	0.89	0.91	0.92
A(12)	0.47	0.51	0.51	0.53	0.58	0.64	0.66
				Japan yields			
mean	0.61	0.67	0.86	1.07	1.52	1.91	2.27
std dev	0.93	0.93	0.99	1.05	1.15	1.21	1.15
min	0.00	0.00	0.01	0.06	0.16	0.27	0.56
max	3.80	3.40	3.58	3.77	4.26	4.59	4.97
A(1)	0.96	0.96	0.96	0.96	0.97	0.97	0.97
A(2)	0.92	0.93	0.93	0.93	0.94	0.94	0.94
A(3)	0.88	0.89	0.88	0.88	0.90	0.91	0.91
A(12)	0.57	0.59	0.60	0.62	0.68	0.72	0.70

Note: "mean" refers to the mean, "std dev" to the standard deviation, and "min", "max" to the minimum and maximum values of the time series of yields for each maturity spanned by the data sample. "A(p)" refers to the autocorrelation of the series at lag "p".

Figure 1 and Table 1 show that US and German data are similar in terms of yield levels and variability over the sampled period. The Japanese data displays much lower levels and corresponding lower variability than the US and German data. A so-called "low yield environment" emerged in the Japanese fixed income markets in the mid 1990'ies and this is clearly seen above. Subsequently, also US and German yields have fallen, however to a lesser degree than what is observed in Japan. Regardless of the evolution in yield levels across the maturities covered by the data sample, all three market segments display high serial autocorrelation. The estimated autocorrelation coefficients are significantly different from zero at a 95% level of confidence for lag one through twelve across all maturities[3]. Such high autocorrelations suggests that the underlying yield series may be I(1). If this is the case we would need to take first-differences to make the variables stationary before valid statistical inference could be drawn, or otherwise resort to co-integration analysis. Economic theory tells us that nominal yield series cannot be I(1), since they have a lower bound support at zero and an upper bound support lower than infinity.

Consequently, and in accordance with the majority of yield curve papers, we model yields in levels and thus disregard that their in-sample properties could indicate otherwise.[4]

3 The modelling framework

In this section we present a coherent model that dynamically evolves yield curves simultaneously in several economic zones. The US is chosen to represent the “cardinal” economy and the German and Japanese yields are modelled as spreads to the US market. Consequently, the yield curve factors determined for the “cardinal” economy affect the evolution of yield factors in the other two economic zones. In other words, the factors driving the evolution of the “cardinal” economy act as common factors for the evolution of yield in all modelled economic areas. Given the large impact of the US economy on the rest of the world it seems reasonable to assume that movements in the US yield curve has a certain effect on the evolution on

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[3] A similar degree of persistence in yield curve data is also reported by Diebold-Li (2006).

[4] It is often the case in yield curve modelling that yields are modelled in levels, see, among others, Nelson-Siegel (1987), Diebold-Li (2006), Diebold-Li-Auroba (2006), Diebold-Li-Yue (2006), Duffee (2006), Ang-Piazzesi (2003), Bansal-Zhou (2002), and Dai-Singleton (2000).

yield curves in other economic zones. In addition to the common factors, idiosyncratic factors affect the evolution of pair-wise yield spreads between the "cardinal" and each of the other countries.

The model is cast in state space form akin to Diebold-Li (2006) and Bernadell et al. (2005). Applying the idea of common and idiosyncratic factors gives the following system:

Observation equation:

$$\begin{bmatrix} Y_t^{us} \\ Y_t^{de} \\ Y_t^{jp} \end{bmatrix} = \begin{bmatrix} H^{C,us} & 0 & 0 \\ H^{C,de} & H^{de-us} & 0 \\ H^{C,jp} & 0 & H^{jp-us} \end{bmatrix} * \begin{bmatrix} b_t^{us} \\ b_t^{de-us} \\ b_t^{jp-us} \end{bmatrix} + e_t, \quad e_t \sim N(0,R) \qquad [1]$$

State equation:

$$\begin{bmatrix} b_t^{us} \\ b_t^{de-us} \\ b_t^{jp-us} \end{bmatrix} = m + F * \begin{bmatrix} b_{t-1}^{us} \\ b_{t-1}^{de-us} \\ b_{t-1}^{jp-us} \end{bmatrix} + v_t, \quad v_t \sim N(0,Q) \qquad [2]$$

The sub-matrices labelled $H^{C,z}$ for z = {us, de-us, jp-us} capture the common effects and follow the Nelson-Siegel (1987) structure: the superscript "C" refers to the term "common". Country superscripts are included on the common matrices for Germany and Japan to allow for the possibility that the number of maturity observations vary between the modelled currency areas – although, this is not the case in the application presented here. Also, the time-decay factor (λ) is allowed to be country specific. The structure of the common matrices is:

$$H^{C,z} = \begin{bmatrix} 1 & \frac{1-\exp\left(-\lambda_z \tau_{z,1}\right)}{\lambda_z \tau_{z,1}} & \frac{1-\exp(-\lambda_z \tau_{z,1})}{\lambda_z \tau_{z,1}} - \exp\left(-\lambda_z \tau_{z,1}\right) \\ 1 & \frac{1-\exp(-\lambda_z \tau_{z,2})}{\lambda_z \tau_{z,2}} & \frac{1-\exp(-\lambda_z \tau_{z,2})}{\lambda_z \tau_{z,2}} - \exp\left(-\lambda_z \tau_{z,2}\right) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-\exp(-\lambda_z \tau_{z,n})}{\lambda_z \tau_{z,n}} & \frac{1-\exp(-\lambda_z \tau_{z,n})}{\lambda_z \tau_{z,n}} - \exp\left(-\lambda_z \tau_{z,n}\right) \end{bmatrix} \quad for\ z = \{us, de, jp\},$$

[3]

where the maturity given by (tau) is denominated in months i.e. $\tau = \{6,\ 12,\ 24,\ 36,\ 60,\ 84,\ 120\}$.

In an effort to gauge how to best approximate the loading structure of the spread curve matrices, i.e. the functional form of H(de-us) and H(jp-us), the eigenvalues are extracted. The data are defined in the following way: for the German market, the difference between yields observed in Germany and US are calculated by: DE-US(t,tau) = DE(t,tau) – US(t,tau) for all time *t* and *tau* (maturities) covered by the data. The Japanese data, JP-US, is defined similarly.

In the spirit of Litterman-Scheinkman (1991) eigenvalues are extracted on the basis of the covariance matrix of each dataset. One interpretation of the eigenvalues is in the form of how much of the total sample variance that is explained by its corresponding factor(s). In particular, let g(j) be the eigenvalue of the jth extracted principal factor of which there is a total K of, then:

$$\begin{pmatrix} \text{explained variance} \\ \text{by principle factor j} \end{pmatrix} = \frac{g(j)}{\sum_{k=1}^{K} g(k)} \qquad \mathrm{j} = 1, 2, ..., K \qquad [4]$$

Table 2 shows the cumulative proportion of explained variance by the principal factors extracted for each spread series. The eigenvalues are ordered in terms of importance from highest numerical value to lowest, and the numerical values reported are the cumulative figures. It can be seen that that two factors explain 97.6% (99.1%) of the variation of German (Japanese) spreads to the US yield curve.

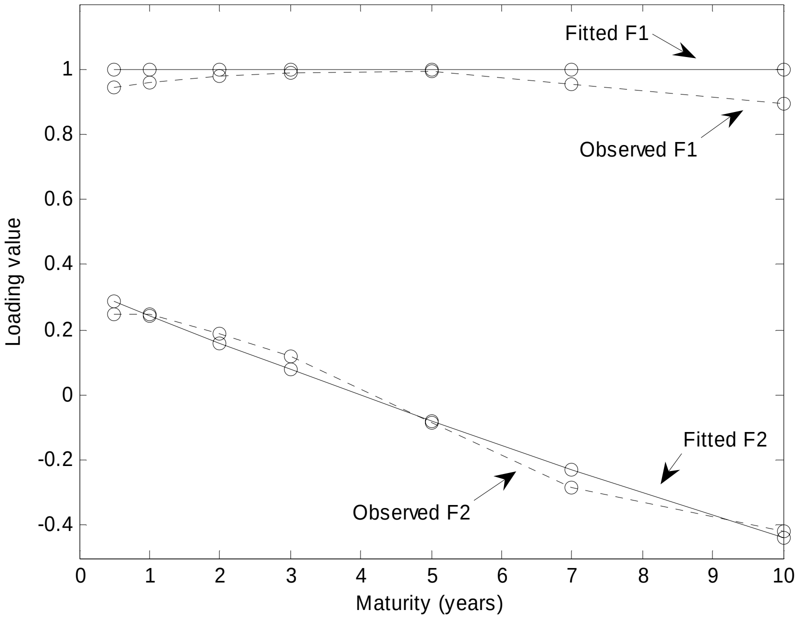
Table 2: Cumulative explained variance of spreads

	Data	
# factors	DE-US	JP-US
1	0.920	0.910
2	0.976	0.991
3	0.994	0.998
4	0.999	0.999
5	1.000	1.000

Note: Cumulative explained variances for the DE-US and JP-US spread data are reported following equation [4].

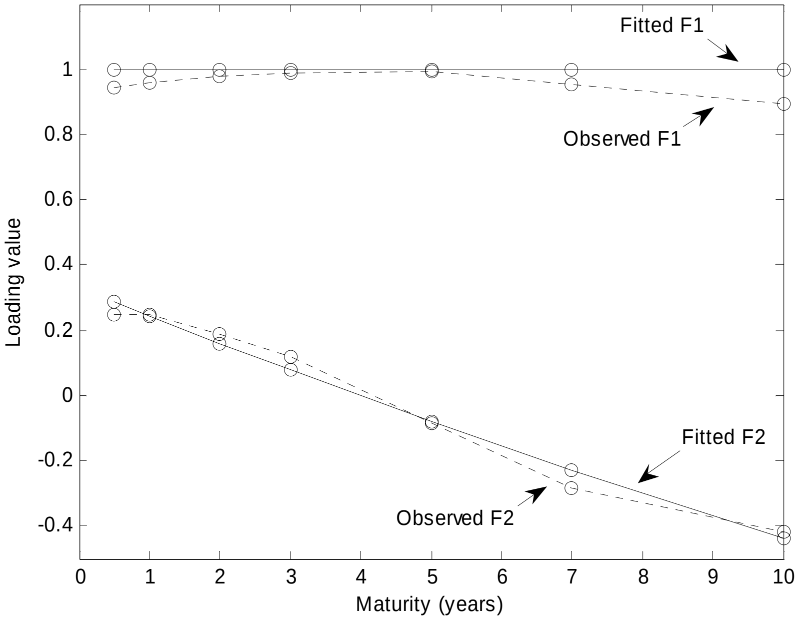
Based on the results shown in Table 2 it seems appropriate to rely on two factors to account for the spread variation in each of the two modelled spread segments. It can be discussed whether three factors should be used in Germany, however, the marginal effect of adding a third factor in Germany is only 1.8% and this is judged to be insignificant in economic terms when contrasted to the extra model complexity that would be implied by having to model a third factor. For the Japanese market it seems clear that only two factors are necessary in order to explain the majority of the variability in the data. Hence, against the background of parsimony we choose to model two factors in Germany and two factors in Japan.

Figure 2: Fitted and observed loading structure for the DE-US spread segment



Note: "F1" refers to factor 1 and "F2" refers to factor 2. The observed loading structure is derived from a factor analysis performed on the DE-US spread data. The fitted loadings follow equations [4] and [5].

Figure 3: Fitted and observed loading structure for the JP-US spread segment



Note: "F1" refers to factor 1 and "F2" refers to factor 2. The observed loading structure is derived from a factor analysis performed on the DE-US spread data. The fitted loadings follow equations [4] and [5].

To approximate the loading structure similar parameterisations are used for both spread data sets. As can be seen in Figure 2 and Figure 3 the first factor has an almost identical impact across all maturities, and, is as such, akin to the Nelson-Siegel level factor. In the context of the loading structure for the spread curves, this first factor is labelled the “shift factor”. The second factor has a negative impact in the short end of the maturity spectrum and a positive effect on longer maturities. Such a pattern gives rise to an interpretation of the second factor as “tilting” the curves around a given maturity observation point. It is observed that the tilt is not linear: the slope of the factor loading seems to decrease throughout the maturity spectrum. As a consequence of these observations the shift factor is approximated by a constant for all maturities, while the tilt factor is modelled by a second order polynomial in the maturity dimension. Therefore:

$$H^s = \begin{bmatrix} 1 & a_s + b_s * \tilde{\tau}_{s,1} + c_s * \tilde{\tau}_{s,1}^2 \\ 1 & a_s + b_s * \tilde{\tau}_{s,2} + c_s * \tilde{\tau}_{s,2}^2 \\ \vdots & \vdots \\ 1 & a_s + b_s * \tilde{\tau}_{s,n} + c_s * \tilde{\tau}_{s,n}^2 \end{bmatrix} \quad for \ \mathrm{s} = \{de - us, jp - us\}, \qquad [5]$$

where, for numerical reasons alone, $\tilde{\tau} = \tau / 12$, i.e. $\tilde{\tau}$ is expressed in years.

Applying the specification in [5] gives the fits for the two spread-factors extracted by principal component analysis, as shown for the German market in Figure 2 and the Japanese market in Figure 3.

4 Model Estimation and validation

Estimation of the state-space model formulated in equations [1] and [2] is done under the assumptions that $e_t \sim N(0,R)$, where $R = \sigma_e^2 I$, and that $v_t \sim N(0,Q)$. The covariance matrices R and Q are assumed to be diagonal with entries containing the variances of the corresponding errors for the maturity segments covered by the yield curve data, and the yield variance of the yield curve factors, respectively. In addition, to lower the number of parameters to estimate, it is assumed that the residual variances in the observation equation are identical across all maturities for the three currency zones modelled.[5] Also, it is assumed that the *F* matrix in the state equation, containing the autoregressive coefficients for the yield curve factors, is diagonal.

The yield curve model has been estimated by the standard Kalman filter technique and parameter estimates are collected in Table 3. It is seen that most parameters are significantly different from zero at a 95% level of confidence. However, this conclusion does not apply to the constant estimates of the state equation [2] i.e. the m-vector. Here only the estimates of the tilt factor in Germany and the spread factor in Japan are significant at the 95% level.

Table 3: Parameter estimates

Observation equation				
Lambda US	**0.080**	**(0.0015)**		
Lambda de-us	**0.053**	**(0.0013)**		
Lambda jp-us	**0.019**	**(0.0009)**		
R	**0.007**	**(0.0002)**		
	DE		JP	
a	**0.323**	**(0.086)**	**0.128**	**(0.031)**
b	**-0.089**	**(0.023)**	**-0.051**	**(0.001)**
c	**0.004**	**(0.001)**	**0.002**	**(0.000)**
State equation				

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[5] Empirically, this constraint seems not to be violated. In fact, loosing degrees of freedom by estimating the additional 20 variance terms, if the constraint is not imposed, seems not to be a worthwhile endeavour on the data used in the current paper.

m_us[1,1]	0.146	(0.0910)
m_us[2,1]	0.001	(0.0274)
m_us[3,1]	-0.025	(0.0407)
m_de[4,1]	-0.024	(0.0139)
m_de[5,1]	**-0.079**	**(0.0358)**
m_jp[6.1]	**-0.097**	**(0.0372)**
m_jp[7,1]	-0.153	(0.0940)
F_us[1,1]	**0.971**	**(0.0159)**
F_us[2,2]	**0.984**	**(0.0123)**
F_us[3,3]	**0.966**	**(0.0162)**
F_de[4,4]	**0.963**	**(0.0138)**
F_de[5,5]	**0.964**	**(0.0095)**
F_jp[6,6]	**0.976**	**(0.0111)**
F_jp[7,7]	**0.967**	**(0.0191)**
Q[1,1]	**0.041**	**(0.0050)**
Q[2,2]	**0.059**	**(0.0074)**
Q[3,3]	**0.156**	**(0.0242)**
Q[4,4]	**0.031**	**(0.0037)**
Q[5,5]	**0.142**	**(0.0692)**
Q[6,6]	**0.051**	**(0.0058)**
Q[7,7]	**0.409**	**(0.0001)**

Note: Estimates for the “observation equation” refer to equation [1]. Estimates for the “State equation” refer to equation [2]. The Indices for the estimated parameters that are reported in hard brackets refer to the element position in the matrices of [1] and [2]. Numbers in bold indicate that the estimate is different from zero at a 95% level of confidence. Standard errors are reported in brackets.

Table 4: Properties of the residuals

	Avg	Stdev	rho(1)	rho(2)	rho(3)	rho(12)
US 6M	-0.01	0.09	0.80	0.62	0.53	-0.03
US 1Y	-0.01	0.05	0.76	0.57	0.50	0.40
US 5Y	-0.03	0.05	0.67	0.47	0.36	0.12
US 10Y	0.00	0.08	0.86	0.75	0.68	0.27
DE 6M	0.00	0.10	0.78	0.68	0.62	0.13
DE 1Y	0.01	0.05	0.73	0.54	0.50	-0.06
DE 5Y	-0.01	0.04	0.77	0.66	0.54	0.18
DE 10Y	0.00	0.06	0.76	0.65	0.55	0.25
JP 6M	0.04	0.08	0.59	0.38	0.22	-0.09
JP 1Y	0.00	0.05	0.65	0.45	0.33	0.00
JP 5Y	0.00	0.08	0.84	0.75	0.64	0.27
JP 10Y	-0.01	0.10	0.87	0.77	0.69	0.34

Note: Summary statistics for the differences between fitted and observed yields (e from equation [1]) are presented. “Avg” is the average of the time series of residual observations, and “std” their standard deviation. rho(p) gives the p’th months autocorrelation.

Table 4 shows the properties of the residuals from the observation equation as given in equation [1]. The conclusions are similar across maturity segments, so for presentational purposes results are only displayed for the segments of 6 months (6M), one year (1Y), five years (5Y) and ten years (10Y). It is seen that on average the residuals have zero mean and display a certain amount of serial autocorrelation. This autocorrelation follows from the fact that the yield curve data is modelled in its levels and as such data is well-known to be highly persistent, it is no surprise that some autocorrelation shows up in the residuals. However, it is also observed that the serial autocorrelation dampens as the lag-length increases, and has almost disappeared at lag 12.[6]

To give a visual impression of the residuals, Figure 4 shows the time series plot for all maturities for each of the modelled markets. A general observation is that most errors are within plus/minus 10 basis points while a few exceptions where maturity segments at specific points in time can diverge by as much as 25 to 50 basis points.

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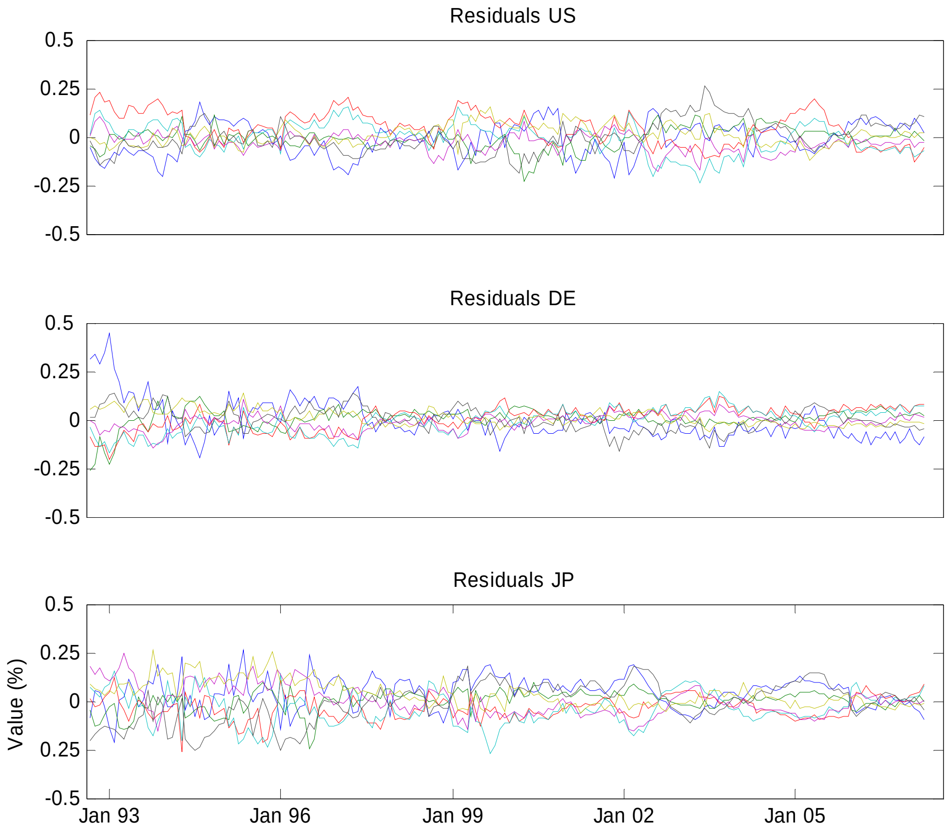
[6] Similar observations regarding the properties of the data are made by e.g. Diebold-Li-Yue (2006).

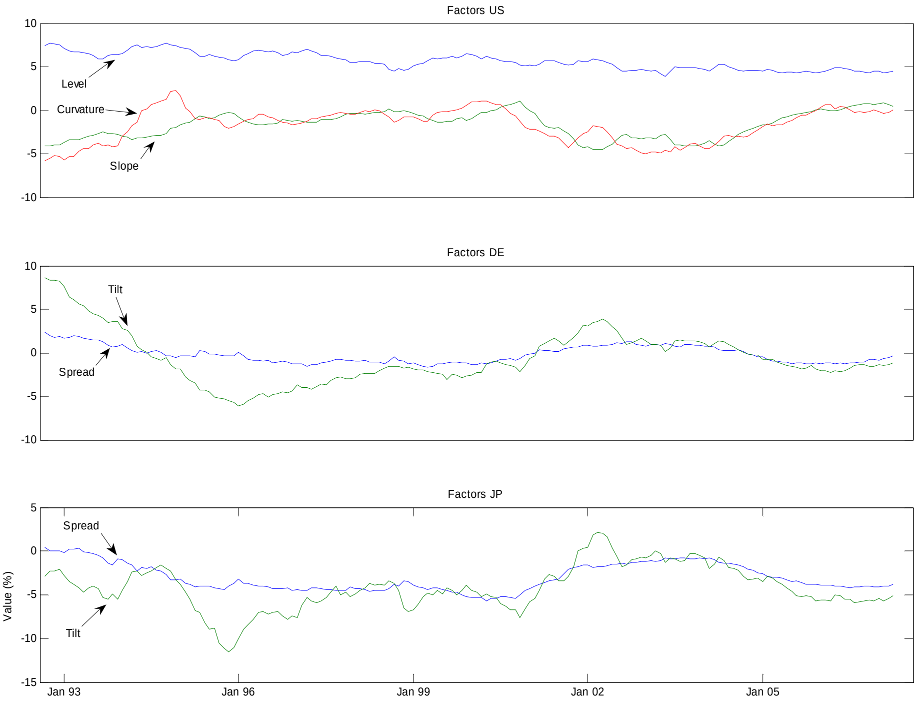
Figure 4: Residuals



Note: the figure shows the time-series of residuals from equation [1].

The time series evolution of the estimated yield curve factors for the US market and the spread and tilt factors for the German and Japanese markets are shown below. Given the model specification the factors extracted from the US market are close in spirit to traditional Nelson-Siegel factors. Due to the applied estimation technique, the spread markets also affect these factors and the results shown for the US market will hence be slightly different from Nelson-Siegel factors extracted from the US market alone. Factors extracted from the spread markets i.e. Germany and Japan, are affected by the US data as well as the data from their specific market segments. However, there are no cross market effects in the sense that the German data does not affect the parameter estimates pertaining to the Japanese market; and the Japanese data does not affect the estimates for the German market. Cross effects arise only via the US market.

Figure 5: Extracted yield curve factors



Note: The figure displays the betas (yield curve factors) from equations [1] and [2].

5 Conclusion

A central building block for risk and portfolio managers who deal with international portfolios is a model that links together the evolution of yields across several currency areas. Such a building block will facilitate the calculation of portfolio risk measures and can serve as a tool for generating return distributions that are correlated across instrument classes and currencies. The current paper shows how a parametric yield curve model can be specified to meet this end.

By treating one market as the “cardinal” currency area, it is shown how the remaining currency area yield curves can be modelled as “spread curves”. Principal component analysis shows that two yield curve factors explain the majority of the observed spread variation. These two factors are interpreted as a spread, that is constant for all maturities at a given point in time, and a tilt that varies across maturities. Both these factors are allowed to change over time. The factor loading structure is shown to be efficiently parameterised by a constant (for the parallel spread) and a second order polynomial in the maturity direction (for the tilt). The cardinal yield curve area is modelled by the Nelson-Siegel (1987) factors.

The model is successfully implemented on yield curve data observed at a monthly frequency for seven maturity observations, covering the period from 1992 to 2007, for the US, German and Japanese markets. The fit of the model is good and estimation errors are in the range of plus/minus 10 basis points.

Given the very flexible modelling setup it would be interesting to investigate if the dynamic evolution of the spread curves can be explained by macro economic variables and exchange rates. Furthermore, the suggested modelling framework could be applied to credit curves within a given currency area with the purpose of identifying how credit-spread movements are related to business cycles.

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