Risk and Return

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Outline

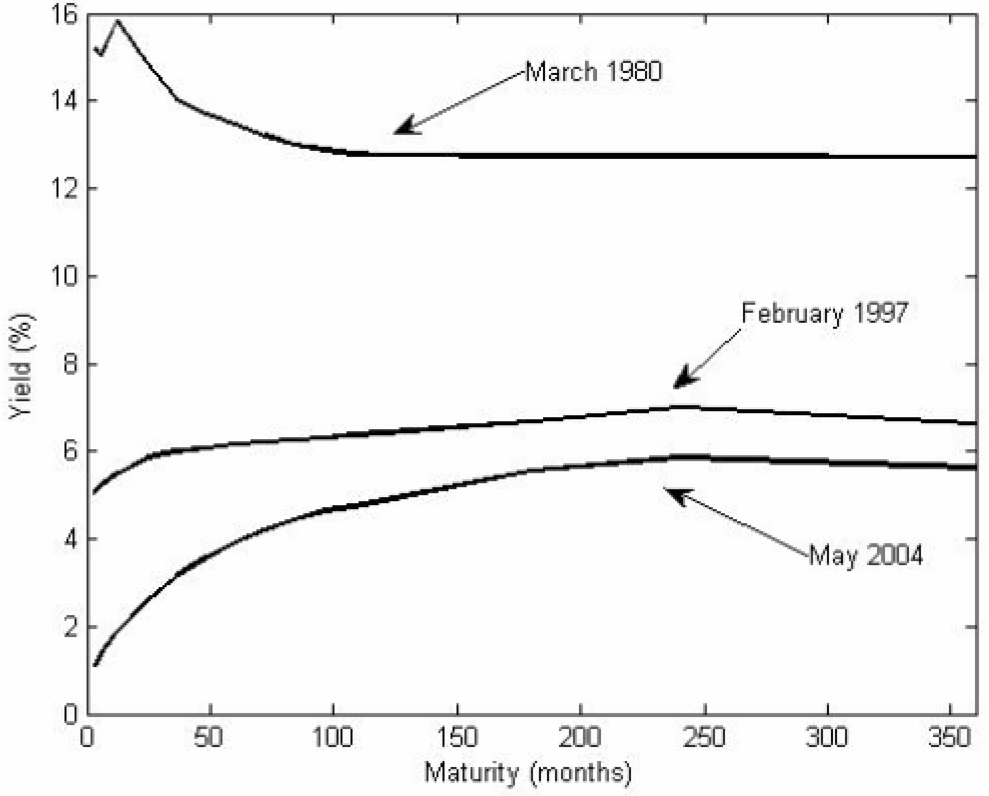
- Rates and yields
- Exercise: ”price2yield” function p.35 in the lecture notes
- Risk measures
  - Variance/covariance matrix
  - VaR, Expected shortfall
  - Modified duration and convexity
- Calculating returns

Rates and yields

- As always in finance the price of an asset is the discounted value of future payments
- The action in bond pricing comes from:
  - Finding the appropriate discount rate
    - The one that matches the risk of the cashflows (credit grade)
    - And, the one that matches the timing of the cashflows
    - And, finally, it is important to remember that the interest rate curve is a dynamic entity

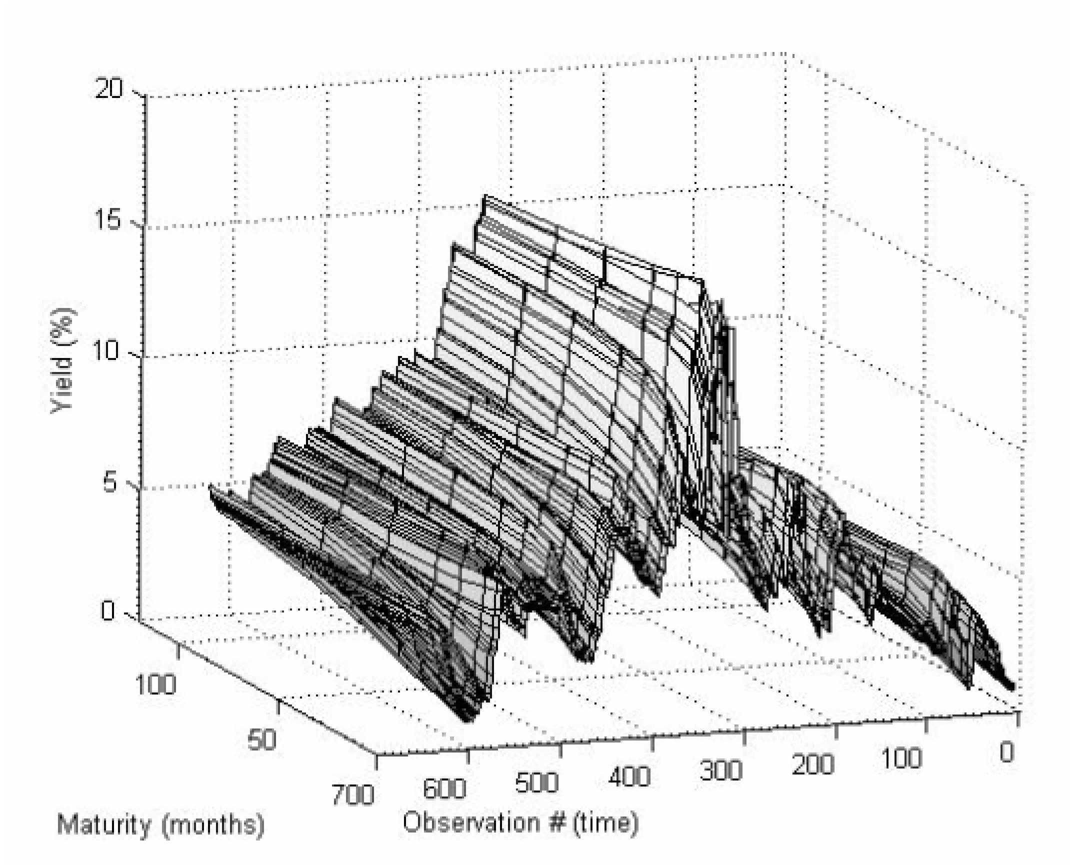
Rates and yields

- Example yield curves



Rates and yields

- Example of an yield curve evolution



Rates and yields

- With zero coupon rate we mean the solution to:

$$P_t = \frac{C}{(1+r_1)^1} + \frac{C}{(1+r_2)^2} + \cdots + \frac{C+100}{(1+r_n)^n} = C^T * D, \qquad (2.1)$$

- With yields we mean the solution to:

$$P_t = \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \cdots + \frac{C+100}{(1+y)^n}. \qquad (2.3)$$

Rates and yields

- Zero coupon rates are the fundamental building block in the fixed income pricing literature
- The are derived from the most liquide bonds in the market
- And used to price other bonds and derivative instruments
- For strategic asset allocation purposes we will argue that it makes sense to look at yields

Rates and yields

- Strategic asset allocation is NOT about:
  - Pricing individual instruments
  - Finding trading oportunities in the market
- Strategic asset allocation is about:
  - Calculating returns for asset class indeces
  - Converting these prices into returns, or
  - Calculating the returns directly via an approximation scheme
- For this yields and yield evolutions are somewhat easier to handle

Exercise

- Replicate the function ”price2yield” on page 35 in the lecture notes and add explanatory text to the function so that your colleagues can understand how the function works
- What kind of function handeling does the code use?

Risk measures

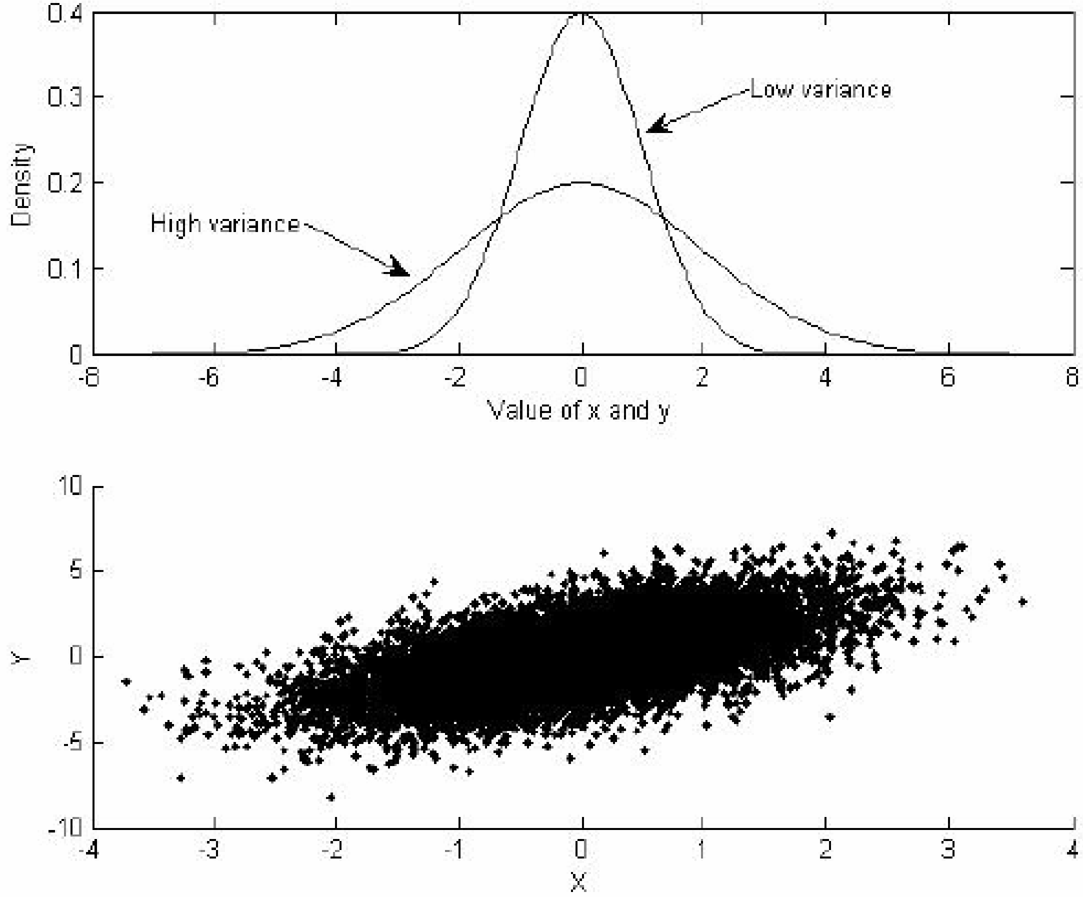
where ρ is the correlation coefficient. The variance and covariance is collectively called the second moment of a distribution and communicates how dispersed the series observations are around the average value and how the two series move together over time. To illustrate this concept assume that the following information is given about x and y:

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim N\left(\ \mu \quad \Omega\ \right),$$

which means that x and y are jointly normally distributed with mean μ and covariance matrix Ω. The covariance matrix has the variances on the diagonal and covariances as the off-diagonal elements. Assume further that

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad \Omega = \begin{bmatrix} 1.00 & 1.2 \\ 1.2 & 4.00 \end{bmatrix}.$$

Risk measures



Risk measures

- Value-at-Risk and Expeceted short fall

$$VaR_\alpha = \min\{l : \Pr[L > l] \leq 1 - \alpha\} = F_L^{-1}(1 - \alpha) \qquad (2.6)$$

$$ES_\alpha = E[L \,|\, L \geq VaR_\alpha]. \qquad (2.7)$$

$$VaR_\alpha = E[r] - \Phi^{-1}(1 - \alpha) * \sigma * \sqrt{t}, \qquad (2.8)$$

- F is the cummulative loss distribution and capital phi is the cummulative normal distribution
- For an example it may be instructive to look at the example on page 41 in the lecture notes

Risk measures

- Modified duration and convexity

  - Bonds:
    - 5Y, 3%
    - 5Y, 15%
    - 10Y, 3%
    - 10Y,15%

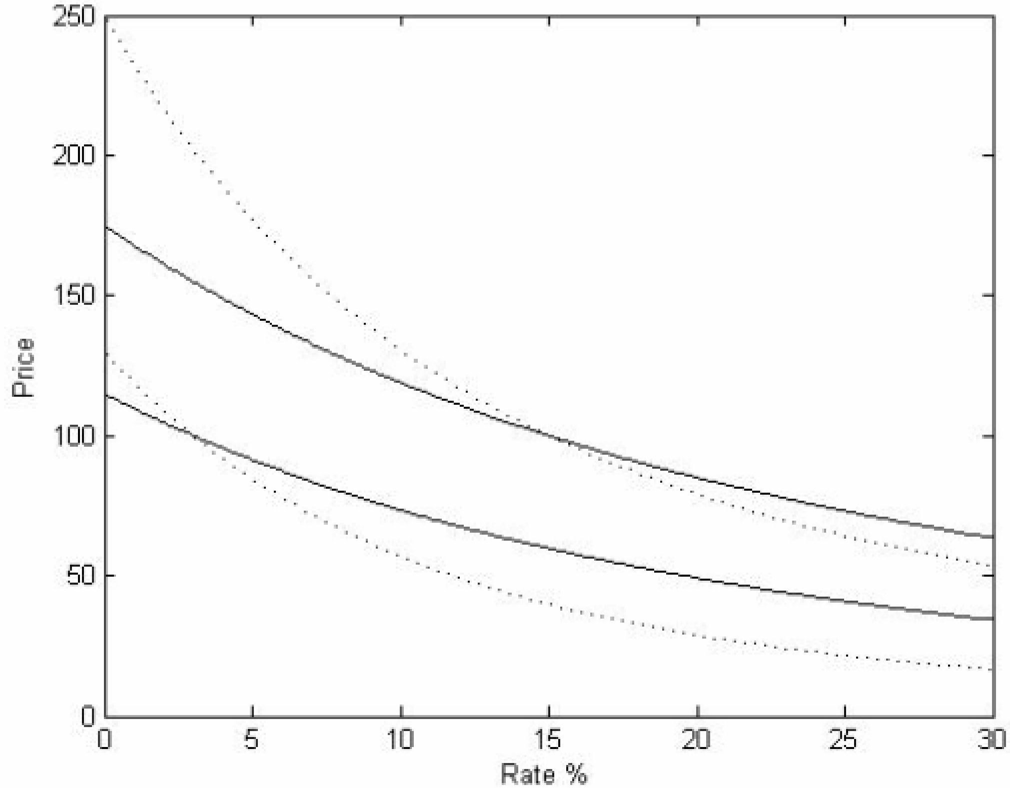


Figure 2.7: Bond price sensitivity to yield levels

Risk measures

- Taylor series expansion of the bond price

$$
\begin{aligned}
P(y+\Delta y) &= P(y) + \frac{dP}{dy} * \Delta y + \frac{1}{2}\frac{d^2P}{dy^2} * \Delta y^2 + R \\
\frac{\Delta P}{P} &\approx \frac{1}{P}\frac{dP}{dy} * \Delta y + \frac{1}{2}\frac{1}{P}\frac{d^2P}{dy^2} * \Delta y^2, \qquad (2.9)
\end{aligned}
$$

- First order (duration)

$$D \equiv -\frac{1}{P}\frac{dP}{dy}, \qquad (2.10)$$

- Second order (convexity)

$$Conv \equiv \frac{1}{P}\frac{d^2P}{dy^2}. \qquad (2.11)$$

Calculating Returns

- The return on equity can be broken down into the following components

$$r^{equity} = \text{change in price} + \text{dividends},$$

- And the return on a bond can be broken down into the following components

$$r^{bond} = \text{change in price} + \text{coupons} + \text{passage of time}.$$

Calculating Returns

Let t denote time and E the expectation operator, then the total return on an asset can be defined by:

$$E\left(r_t^{tot}\right) = \frac{\text{future amount received}}{\text{amount paid}} = \frac{E\left(P_t\right)}{P_{t-1}} \tag{2.18}$$

and the relative return can be defined as:

$$\begin{aligned} E\left(r_t\right) &= \frac{\text{future amount received - amount paid}}{\text{amount paid}} \\ &= \frac{E\left(P_t\right) - P_{t-1}}{P_{t-1}} = \frac{E\left(\Delta P_t\right)}{P_{t-1}}. \end{aligned} \tag{2.19}$$

It is clear that:

$$E\left(r_t\right) = \frac{E\left(P_t\right)}{P_t} - \frac{P_{t-1}}{P_{t-1}} = E\left(r_t^{tot}\right) - 1$$

and that:

$$E\left(P_t\right) = \left[1 + E\left(r_t\right)\right] * P_{t-1}.$$

A continuos compounding version of the above return calculation also exists. According to this the return is calculated by:

$$E\left(r_t\right) = \ln\left(\frac{P_t}{P_{t-1}}\right).$$

Calculating Returns

- Taking into account
  - The passage of time
  - The change in price due to interest rate changes
  - The coupon
- The return on a bond can be calculated by

$$r_{t,t+j}^{bond} = \frac{P_{t+j}^{(n-j)} - P_t^n + C_t}{P_t^n}. \qquad (2.20)$$

- Where *n* is maturity, *t* is time, *P* is price, *C* is coupon, *j* the length of time the bond is held